

## Application of interpolation procedures for presentation of data electromagnetic wave lightening

I. M. Aleshin<sup>1</sup> and V. M. Zhandalinov<sup>2</sup>

Received 28 October 2009; accepted 14 November 2009; published 20 November 2009.

Electromagnetic wave lightening technique is an important geophysical tool in geological prospecting, including kimberlitic tube detection. Development of transmitters and receivers with the minimal geometrical characteristics allows to process measurements practically on over all well. Processing such data permits us to construct three-dimensional model of site of investigation. In the report one of possible techniques of construction of such model, based on interpolation procedure is presented. **KEYWORDS:** *electromagnetic wave lightening, interpolation, inverse distance method, 3D model.*

**Citation:** Aleshin, I. M. and V. M. Zhandalinov (2009), Application of interpolation procedures for presentation of data electromagnetic wave lightening, *Russ. J. Earth. Sci.*, 11, ES1005, doi:10.2205/2009ES000430.

### Introduction

This article describes some aspects of radio wave measurement interpretation, when both source and receiver are placed in drilled wells. Current interpretation of such a measurements based on homogeneous media model [Petrovskiy, 2001]. The model implies source electric field  $E$  decreases with distance  $R$  by formula

$$E = E_0 \frac{e^{-\kappa R}}{R}. \quad (1)$$

Here  $E$  - measured electric field,  $E_0$  - so-called “initial field” - is a constant, derived from data,  $\kappa$  - absorption coefficient, which connected with conductivity  $\sigma$  via relation

$$\sigma = 2\kappa(\varepsilon\varepsilon_0/\mu_0 + \kappa^2/\omega^2\mu_0^2)^{1/2},$$

where  $\omega$  is circle frequency,  $\varepsilon$  is a rock dielectric constant,  $\varepsilon_0$  and  $\mu_0$  are vacuum electrical and magnetic constants, correspondingly.

Naturally, on practice we have more or less expressed trend which allows more likely to talk about tendency than about exact conformity to the formula (1) (see Figure 1). Existence of such trend allows to define “initial field”  $E_0$  (at the point of the transmitter’s location) and the average absorption factor  $\kappa$  on the site.

When  $E_0$  is defined, it is possible to calculate effective fading factor for each measurement:

$$\kappa = -\ln \frac{E_0}{ER},$$

which characterizes an average value of this parameter in corresponded space between the wells. It has to be noted that on this stage we face contradiction with the used homogeneous model. In fact, we assume that there is a possibility to describe the electric field by formula (1), but with absorption coefficient which depends on the distance:  $\kappa = \kappa(R)$ . In order to prove this assumption we need to place our initial-state manifold into Maxwell’s equations in order to find the conditions when the elements which are different from the right side of the formula (1) can be neglected. In terms of mathematics such approach is difficult. Instead we can offer easy and qualitative method to estimate the validity of homogeneous model. It’s based on fact that in absorption coefficient definition dependence from an electric field is defined by “slow” logarithm. Therefore during absorption coefficient distribution analysis there is a need to pay direct attention to the relation

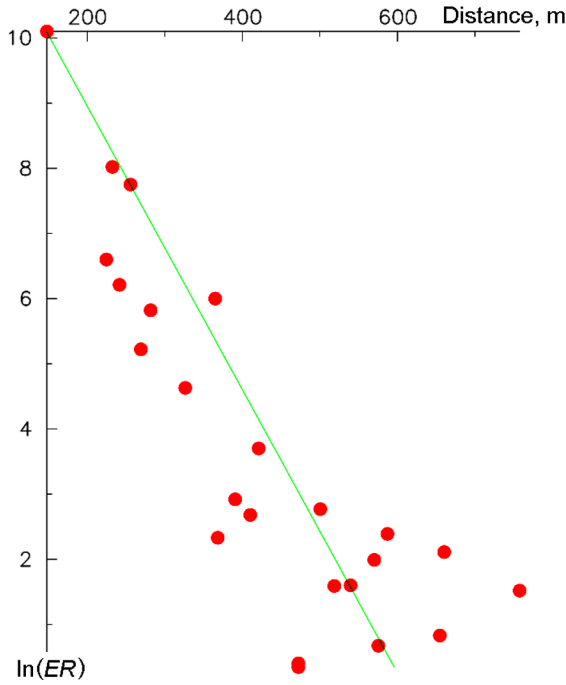
$$\frac{1}{S} = \frac{E}{E_0 e^{-\kappa R}},$$

where  $S$  is the so-called screening coefficient. Abnormal values show significant deviations from the assumed uniformity of the model as well as an existence of non-uniform parts between corresponding wells.

It is important to remember that described procedure of data processing is better when the size of the site is small,

<sup>1</sup>Institute of Physics of the Earth RAS, Moscow, Russia

<sup>2</sup>AC “ALROSA”, Moscow, Russia



**Figure 1.** Dependence of value  $\ln(ER)$  from the distance for measurements on “Kyllachsky” site.

because increasing of it’s size also increases probability of non-uniformity which decrease usefulness of approximation of homogeneous media.

### A Scheme of Construction of Horizontal Slices. Space Averaging

In a history of evolution of this method, significant period of time was taken for developing small sized transmitting and receiving antennas, which allowed to perform measurements on depths with small intervals and provide high level of discontinuity and informative of the measurements and also optimization of financial expenses for working intervals wells drilling. Therefore the important feature of radio wave measurements is the interval discontinuity and results grouping by depth intervals (vertical columns). Such character of the data is caused by the binding with the wells or similar to the binding to the locations of seismostation in the receiver function tomography method. [Vinnik *et al.*, 2006; Kozlovskaya *et al.*, 2008].

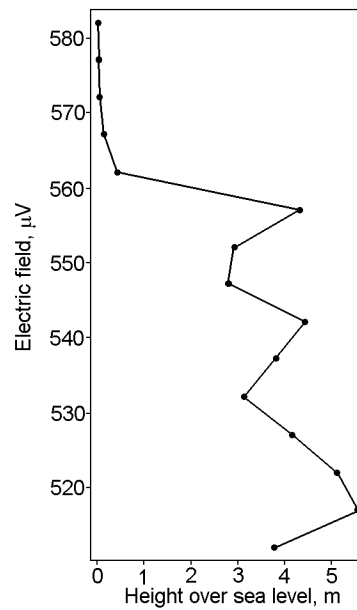
Moreover it is important that even when largest possible measurement’s step is chosen, registered field is not smooth (Figure 2). This caused by rock inhomogeneity both in vertical direction and on lateral. To avoid the influence of aliasing in construction of the 3-D model [Mitchell and Netravali, 1988] the initial data has to be smoothed. The easiest way to smooth is to average of measurements in the layer. The obvious way to select the layer is the selection of so-called depths “working interval”. The last one is selected on each

well by the value of measured electric field. The average value of the field within this interval is used for interpretation. Such choice of averaging is physically justified, but very rough. In particular, it does not allow to receive the picture of changing of studied properties of the slice with depth. The formal choice of the layers can be an alternative which would be irrelative of the physical properties of the rock from one well to another. Such smoothing allows to develop the series of maps of absorption coefficient distribution in horizontal slices of the site for different depths and to track changes of rock’s properties in the area.

Formally the procedure for developing of such maps can be defined in the following way. First, we look through all measurements and define  $H_{min}$  and  $H_{max}$  – minimum and maximum height above sea level, accordingly, for which measured data happens to be. Here, this is the point: because of relief of the site and other circumstances, measurements conducted in different wells can start from different depths. Therefore in all the wells data happens to be only from some high above sea level, which we marked as  $H_{max}$ . Similar way,  $H_{min}$ , in fact the depth of the last count in the measurements of the most “shallow” well. Knowing  $H_{min}$  and  $H_{max}$ , it is possible to define and amount of the slices  $N_s$ :

$$N_s = [(H_{min} + H_{max})/\Delta H] - 1,$$

where  $\Delta H$  - layer’s thickness. The square brackets mean that the fractional part has to be neglected. Each of the slices will be referred to the height of the middle of the layer. The value  $\Delta H$  has to be chosen in a way that from one hand the layer could contain enough counts for averaging, on another hand their amount would allow to track changes of site properties connected to depth.



**Figure 2.** Example of radio wave measurements – dependence of electric field ( $\mu V$ , horizontal axis) from the height above sea level (m, vertical axis). Absolute point of well surface – 607 m.

In order to draw each of the slices, the following operations are executed for particular layer: for each measurement  $E$  is an average value of electric field on a layer has to be found; by calculated value of the average field, the average square factor  $E_0$ , is defined, which is needed in order to find fading factor  $\kappa''$  for each measurement according to the formula:

$$\kappa'' = -\ln \frac{ER}{E_0},$$

Therefore we form the main input array which will be used to calculate the slice: set of three  $x,y,\kappa''$ , where  $x, y$  are the coordinates of the middle of the segment which connects to the transmitter and receiver on which one of interpolation schemes, described in the following section, is applied.

Continuous site data filtration can be as an alternative for the layer by layer averaging [Kozlovskaya et al., 2008]. Generally speaking, the degree and method of smoothing depend on detalization degree of model, because vertical resolution has to be coordinated with lateral resolution.

### Interpolation Procedure

Let's draw our attention to the question of analysis of absorption coefficient distribution between wells using the results of field researches. Due to the fact that the Measurements give us the averaged values of the coefficient, so the task can be formulated in the following way. Let's assume that we have to find distribution of some value  $K = K(\mathbf{r})$ , and we have measured values in the limited number of points  $\mathbf{r}_i$ . It is necessary, using these data, to find needed value in any arbitrary point  $\mathbf{r}_i$ .

### Ray Tomography Method

Let's look at the ray tomography method. The idea of using it in geophysical applications comes from seismology [Nolet, 2008]. Let's look at the site where  $N$  measurements were performed. Then split the site on  $M$  cells. Let's assume that fading coefficient inside the cell does not change, but during the transition from one cell to another, it can change, generally speaking. Then for each  $j$ -th ray we have

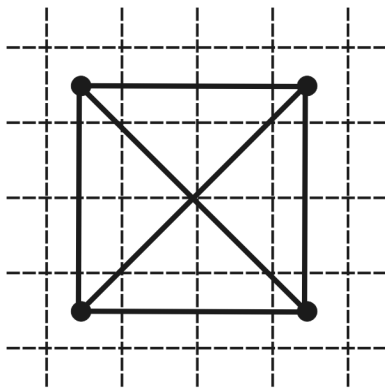


Figure 3. Splitting on cells.

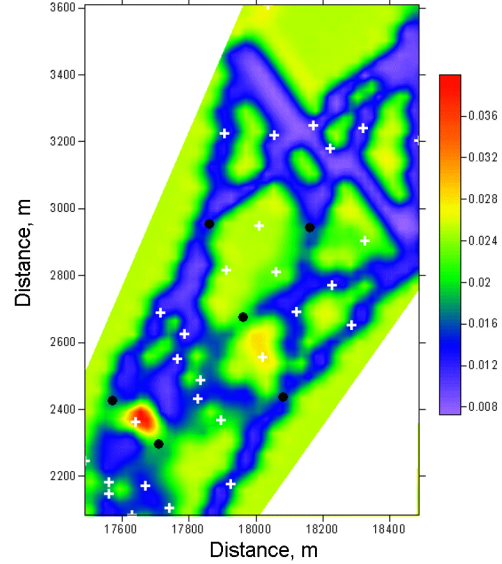


Figure 4. Tomographic calculations result.

an equation

$$\sum_{i=1}^{L_j} dk_i^j dl_i^j = \kappa_j R_j.$$

After solving the system of such equations, it's possible to find the searched values in each cell. However in case of radio wave lightening this method cannot provide needed results. Let's evaluate the site with four wells where six possible measurements has been conducted (Figure 3).

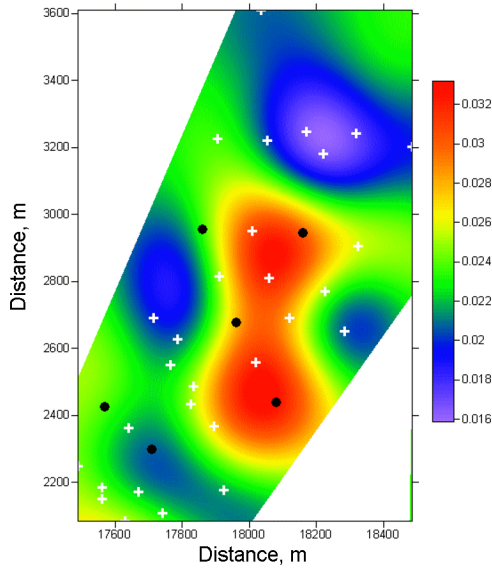
Minimal practically interesting amount of cells needs to be equal 16, but the number of measurements in our case equals 6. Mathematically this means that we have significantly indefinite system of linear equations: 6 equations for 16 indefinite values. Further increasing of amount of cells aggravates the situation even more, because the appearance of new cells through which no ray is going through. An example of such use of ray tomography method on data from "Kyllachsky" site is shown below (Figure 4).

### Interpolation

Another approach to calculate the distribution of properties of interest is the interpolation procedure. In a general, linear evaluation of value  $K(\mathbf{r})$  in the point  $\mathbf{r}$  can be found using measurements performed in  $L$  and other points according to the formula:

$$K(\mathbf{r}) = \sum_{i=1}^L w_i(\mathbf{r})\kappa_i,$$

where  $w_i(\mathbf{r})$  are weighting factors. The values of this weights are defined by the particular procedure of interpolation. In modern geological and geophysical researches kriging method is widely used [Kriging, 1951] - method which was called after the last name of it's creator. In this method the set of measurements is discussed as realization of random process. Measurements analysis allows to draw statistical data model



**Figure 5.** Kriging method interpolation result.

(correlation factor), which can be used during weight calculation. Figure 5 illustrates distribution, received by kriging method for the same data like on previous figure.

From a few lacks of kriging, two needs to be mentioned. The first one – significant volume of measurements. Having great amount of points or need to perform calculations on private net, these measurements can take long time period even if modern computers are used. The second lack of kriging – need for prior data research in order to find the statistical model and it’s parameters. If forces us to consider one more interpolation procedure [Liszka, 1984], which is the result of development of simple method of inverse distances. In this procedure weighting factors are calculated by the formula

$$w_i(\mathbf{r}) = \frac{1}{W} (d(\mathbf{r}, \mathbf{r}_i) + \eta)^{1/2}.$$

Here  $d(\mathbf{r}, \mathbf{r}_i)$  is the distance between the points  $\mathbf{r}$  and  $\mathbf{r}_i$ ,  $W$  – normalization coefficient, which is found from the condition

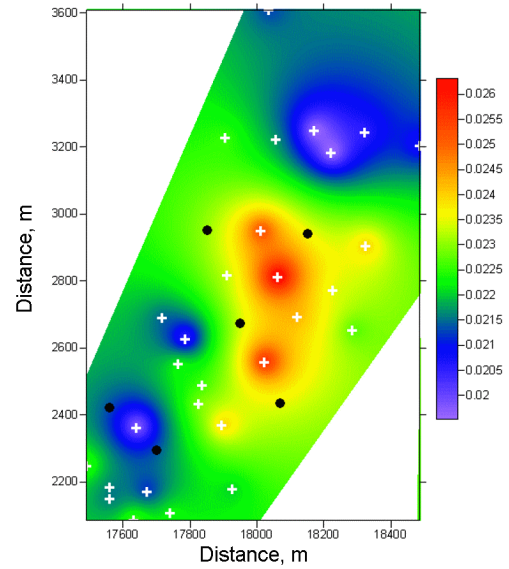
$$\sum_{i=1}^L w_i(\mathbf{r}) = 1.$$

The presence of the only parameter  $\eta$ , which is defined generally speaking from accuracy of data allows us, on one hand, to take into account inaccuracy of model assumed and, on another hand, to avoid an effect of “bull eye”, which inherent the initial method of inverse distances.

Results of application of this method to the same data from “Kyllachsky” site is shown on Figure 6.

## Conclusion

In the conclusion we formulate the main points: physically proved calculation of distribution of the main studied characteristics is possible if complex environment models are



**Figure 6.** Modified method of inverse distances result.

used. Applicability of tomography method for radio lightning data is limited. The most effective method of tomography like picture is modified method of return squares. This method is more simple and more effective at the same time. Interpolation procedure realization is possible only for filtered data. Simultaneously with visualization of topographic slice there is a need to bring the numeric values of screening factors.

## References

- Vinnik, L. P., I. M. Aleshin, M. K. Kaban, S. G. Kiselev, G. L. Kosarev, S. I. Oreshin, K. Raiber (2006), Kora I mantiya Tyan-Shanya po dannym prienyh funktsij, *Fizika Zemli*, 14.
- Petrovskiy, A. D. (1971), *Radiovolnovye metody v podzemnoy geofizike*, Nedra, Moscow.
- Krige, Danie G. (1951), A statistical approach to some basic mine valuation problems on the Witwatersrand, *J. of the Chem., Metal. and Mining Soc. of South Africa*, 52(6), 119.
- Kozlovskaya, E., G. L. Kosarev, I. M. Aleshin, O. Yu. Riznichenko, I. A. Sanina (2008), Structure and composition of the crust and upper mantle of the Archean-Proterozoic boundary in the Fennoscandian Shield obtained by joint inversion of receiver function and surface wave phase velocity of recording of the SVEKALAPKO array, *Geophys. J. Int.*, 175, 135.
- Liszka, T. (1984), An interpolation method for an irregular net of nodes, *International Journal for Numerical Methods in Engineering*, 20(9), 1599.
- Mitchell, Don P., Arun N. Netravali (1988), Reconstruction filters in computer-graphics, *ACM SIGGRAPH International Conference on Computer Graphics and Interactive Techniques*, 22, 1.
- Nolet, G. (2008), *A Breviary of Seismic Tomography*, University Press, Cambridge.

I. M. Aleshin, Institute of Physics of the Earth RAS, Moscow, Russia (ima@ifz.ru)

V. M. Zhandalinov, AC “ALROSA”, Moscow, Russia