Models of generation of power laws of distribution in the processes of seismicity and in formation of oil fields and ore deposits

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[1] A feature indicative of non-equilibrium dynamic system is a fulfillment of a power law of distribution. We consider a number of simple models of generation of power-law distributions in seismic processes and in the processes of mineral deposits formation. As for seismicity, it is shown that deterministic earthquake prediction does not appear to be possible within the used model, whereas a probabilistic prognosis is feasible. Thus it seems possible to reconcile the well-known theoretical concept of unpredictability of earthquakes with the actual existence of algorithms realizing a non-trivial prognosis of strong earthquakes. As for processes of mineral deposits formation, models are discussed that permit to explain a generation of a power-law distribution of a number of large hydrocarbon and ore deposits from the stock values. A new interpretation is proposed of relatively less (than could be expected from the extrapolation of a power distribution law) number of deposits with relatively smaller stock values. *INDEX TERMS:* 0545 Computational Geophysics: Modeling; 3245 Mathematical Geophysics: Probabilistic forecasting; 3265 Mathematical Geophysics: Stochastic processes; 3665 Mineralogy and Petrology: Mineral occurrences and deposits; 4415 Nonlinear Geophysics: Cascades; 4468 Nonlinear Geophysics: Probability distributions, heavy and fat-tailed; 7223 Seismology: Earthquake interaction, forecasting, and prediction; *KEYWORDS:* Power laws, dynamic systems, earthquake prognosis, deposit size distribution.

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Introduction

[2] A characteristic feature of dynamic system is the fulfillment of a power law of distribution,that corresponds to a high degree of concentration of a total effect in a small number of a few strongest events. Guttenberg-Richter earthquakes recurrence law is a well known example of the power law distribution of a number of earthquakes from their energy and seismic moment values. Power distributions are frequently associated with fractal geometry of corresponding objects. They are abundant in nature and are typical of dynamic systems of different physical nature. The distribution of blocks number (tectonic plates) in relation to their size, the distribution of deposit number in relation to the stock values, self-similar character of the geometry of micro-crack structure in samples and tectonic faults in the lithosphere and others are examples that can be treated as power-law distributions [*Bak et al.,* 1987, 1988; *Burshtein,* 2006; *Czechowski,* 2003; *Kontorovich et al.,* 1985; *Mandelbrot,* 1982; *Sadovskiy,* 1989; *Sornette,* 2000; *Sornette and Pisarenko,* 2003; *Turcotte,* 1997; and others] .

[3] Realization of self-similar power distributions in geophysical systems is treated ordinary by analogy with the critical type of behavior [*Bak et al.,* 1987, 1988; *Ito and Matsuzaki,* 1990; *Klimontovich,* 1995; *Sornette,* 2000; *Tyupkin,* 2007; *Zaliapin et al.,* 2002; and others]. It is corroborated by similarity noted in the behavior of such systems with properties occurring in the vicinity of critical points and in the vicinity of second order phase transitions, since power laws describe the variation of physical characteristics in the vicinity of critical points [*Ma,* 1980; and others]. Such behavior occurring in critical points and during second order phase transitions is explained by the growth of fluctua-

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tions (their magnitude and spatial scale) as the critical point (phase transition) is approaching and individual elements of the system behave in a more and more cooperative way.

[4] The analogy between the process of seismogenesis and second order phase transition is widely used in modern geophysics to understand the nature of seismic process and in elaboration of algorithms of earthquake prognosis [*Ito and Matsuzaki,* 1990; *Tyupkin,* 2002, 2007; *Zaliapin et al.,* 2002; and others]. The effect of increasing of seismic regime correlation, which by this analogy may be expected during strong earthquake preparation, was revealed from the results of earthquake catalog studies and in model experiments. This effect manifests itself in the following features: in an increase of in a number of swarms of earthquakes and in appearance of remote earthquakes and earthquake chains. Indicators of such increase in correlation of seismic regime were used in elaboration of new algorithms of prognosis of strong earthquakes [*Rundle et el.,* 2000; *Sobolev and Ponomarev,* 2003; *Tyupkin and Giovambattista,* 2005; *Zaliapin et al.,* 2002; and others].

[5] However in the use of the analogy between seismic regime and second order phase transition, the important differences between these two phenomena should be taken into account. These differences may be reduced to two major moments. Firstly, geophysical systems and specifically seismic regime are described by power self-similar laws everywhere and not only in the vicinity of some special points as in the case of critical phenomena and phase transitions. This difference is minimized in the model of self-organized criticality, the SOC-conception. The SOC model describes and even partly postulates the process of spontaneous evolution of a complex system in the direction of developing of a self-similarity [*Bak et al.,* 1987, 1988]. Secondly, the critical phenomena and second order phase transitions (as distinct from first order phase transitions) occur without energy release (or absorption). But geophysical self-similar processes, specifically earthquakes commonly release large amount of energy stored up in the system before.

[6] The differences noted above are significant. Therefore, besides the SOC model, other models that could describe the origin of self-similar power distributions in geophysical systems are to be proposed. Some versions are given in [*Czechowski,* 2001, 2003]. Below, we discuss the approach based on the model of power law distribution appearing in a result of a set of episodes of avalanche-like relaxation of a set of formed before metastable situations (metastable subsystems) [*Rodkin,* 2001]. This model formally corresponds to a multiplicative cascade model was used before in [*Rodkin,* 2002b] to model a regime of natural disasters. In this paper, we apply this model to describe a seismic process and origin of power-law distribution in seismic process and to model the empirical distribution of a number of deposits from the amount of resources [*Burshtein,* 2006; *Kontorovich et al.,* 1985; *Sornette,* 2000; *Turcotte,* 1997]. This model [*Rodkin,* 2001, 2002b] allows us to advance to a better insight in the variety of conditions in which power law distributions can be formed, in conditions of realizing of deterministic and/or statistical prediction of strong earthquakes, and in processes of formation of empirical distributions of number of deposits in relation to their resources.

Description of Seismic Regime With the Use of Multiplicative Cascade Model. Probabilistic and Deterministic Prognosis of Earthquakes

[7] Distribution of a number of earthquakes of different size (seismic moment and/or seismic energy values) obeys the Guttenberg-Richter law of earthquake recurrence. Different authors discussed the nature of recurrence law and besides the prevailing now SOC-concept other approaches to the problem were proposed also [*Golitsyn,* 2001; *Grigorian,* 1988; *Rodkin,* 2001; and others]. One of the models formulated in a general way corresponds to an occurrence of a power distribution in a result of a large number of episodes of development of stochastic avalanche-like processes when the velocity of process increase is statistically proportional to its current value [*Rodkin,* 2001].

$$\dot{x} = kx \ , \tag{1}$$

where k is a random value with positive mean value, and the avalanche-like process (1) at each step may continue with probability p or to be interrupted with probability $(1-p)$.

[8] It can be easily shown that the set of values X_i obtained as a result of a series of processes (1) will be distributed according to a power law. In fact, solving (1), we obtain values of individual events x, which realize in the result of n steps of the process:

$$x = x_0 \exp(an\Delta t) \ , \tag{2}$$

where x_0 is the initial value, n is a step number, Δt is a step length. The probability of process interruption at step number n and accordingly of formation of event of value x is equal to

$$P(t = n\Delta t) = (1-p)p^n \ . \tag{3}$$

[9] Thus we obtain

$$P\big(x_0 \exp(an\Delta t) \geq x\big) = p^n \ , \tag{4}$$

where infinite geometric progression is summed up:

$$(1-p)p^n + (1-p)p^{n+1} + \ldots + (1-p)p^{\alpha} \ .$$

From (4) it follows

$$\begin{aligned} \ln\big\{P(x_0 \exp(an\Delta t) \geq x)\big\} &= \ln\big(1 - F(x)\big) \\ &= \big\{\ln(p)/(a\Delta t)\big\} \times \ln(x/x_0) \ . \end{aligned} \tag{5}$$

[10] In transition to continuous process that may be interrupted with equal probability at any arbitrary time moment, probability p of process development continuation at an arbitrary Δt may be written as $p_0^{\Delta t}$, where p_0 is a probability of continuation of process development for the step of single-unit duration. Taking it into account we obtain from (5)

$$\ln\big(1 - F(x)\big) = \big\{\ln(p_0)/a\big\} \times \ln(x/x_0) \ . \tag{6}$$

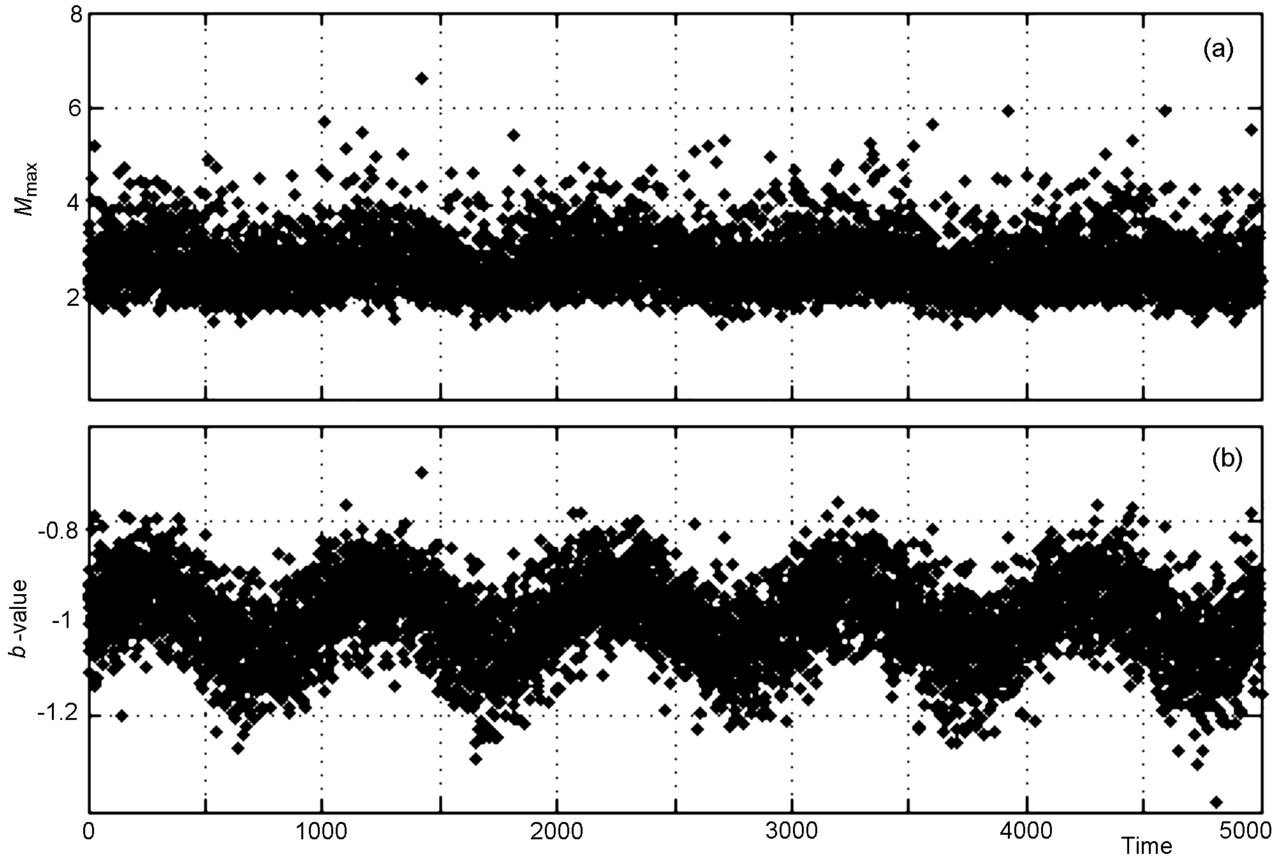

Figure 1. An example of typical realization of seismic regime stochastic model by scheme (1). Maximum values of magnitudes are given $M_{\max}$ (a) and recurrence plot slope β-values (b) are given for sequential time intervals.

[11] From (6) it can be seen that model (1) leads to a power-law distribution of a number of events in relation to their size x.

[12] As applied to earthquake model, let us imagine seismic process as a set of episodes of avalanche-like relaxation of elastic energy accumulated before (or relaxation of inner energy of rocks, for example of energy of metastable mineral assemblies). Characteristics of such model are the intensity of events flow N and two parameters: mean values of parameter k and probability of cessation of an avalanche-like process in time unit p. Parameters k and p in combination determine the slope of the recurrence plot β of a number of events from their values X_i (magnitudes of "earthquakes") in double-logarithmic coordinates

$$\beta = \left\{\ln(1/p)\right\}/\left\{\ln(1+k)\right\} . \tag{7}$$

[13] Using (7), it is easy to select values of parameters k and p, with which the obtained values of slope of recurrence plot β and magnitude m (for example $m = \lg(X_i)$) correspond to values typical of seismic process. Specifically, if we assume initial values of X_i equal to one and mean values $p = 0.5$ and $k = 1$, then we obtain the recurrence plot slope $\beta = 1$.

[14] If we preset the average number N of such avalanche-like processes in time unit and a certain regularity of change of parameters k and p with time, the model in question will produce a sequence of values of model magnitudes of events $\lg(X_i)$ similar to the earthquake magnitudes in an actual seismic process.

[15] To model change in seismic process we preset weak periodic variations in recurrence plot slope β (with amplitude 0.2 and period equal to 1000 time intervals) and assume seismic flow intensity to be $N = 500$ events in unit time. In Figure 1, a typical realization of a model process in a time section as long as 5000 conventional time intervals is given. The obtained this way change of maximum values of magnitude $M_{\max}(t)$ and recurrence plot slope β-value(t) are similar to typical values of an actual seismic process (apart from preset of a periodic character of variation of recurrence plot slope β with time).

[16] The discussed simple and purely stochastic model produces a well-known "prognostic" indicator, that is time intervals of strong earthquake occurrence $M_{\max}$ are preceded in average by decrease in recurrence plot slope β-values. To show this dependence clear in Figure 2 the plot is shown of magnitude maximum values $M_{\max} = \lg(X_i)$ related to values of recurrence plot slope β in the time interval preceding the time interval to define $M_{\max}$. As it can be seen in Figure 2 these parameters are correlated: peak events $M_{\max}$

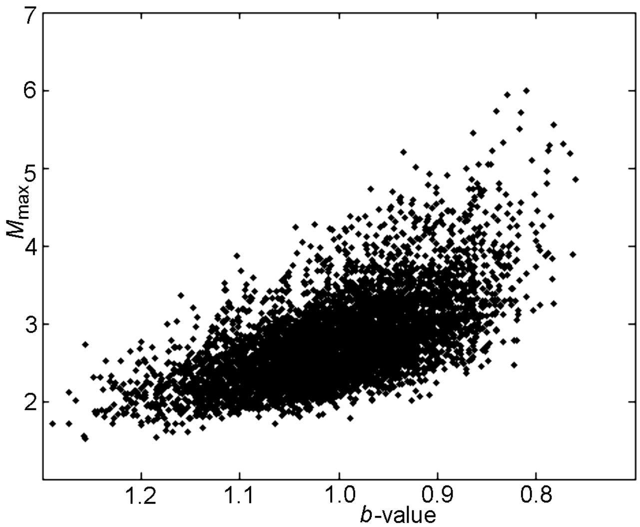

Figure 2. Model relationship between recurrence plot slope β-value and maximum amplitude $M_{\max}$ reached in subsequent time interval. Increased probability of strong event occurrence can be seen in time intervals preceded by decrease in β-value.

are realized (statistically) with relatively lower preceding recurrence plot β-value. The mechanism that causes such a correlation is clear. Indeed, generation of earthquake with large magnitude value corresponds statistically to the values of p and k parameters that at the same time correspond to lower values of recurrence plot slope β-value.

[17] It should be emphasized that in the model under consideration the effect of decrease in the β-value is not a prognostic indicator of a strong event being prepared (it is not correct to speak about strong event preparation as applied to a sequence of independent events) but a parameter related to the probability of occurrence of a strong event. Statistically such an anomaly is characteristic of a time interval before, during and after the occurrence of the strong event. Interpreting the obtained result as applied to the problem of prognosis of strong earthquakes, we obtain that statistical prognosis of strong event occurrence is possible but this prognosis has a stochastic character. Each individual event is a random phenomenon.

[18] Similar situation may take place in the case of actual seismic regime. In this case, the prediction suitable for practical use may be possible but not in the sense it is commonly understood. In terms of the used model the time intervals when the probability of strong earthquake occurrence is greater can be indicated whereas the physical "process of strong earthquake preparation" as such is absent. This situation differs essentially from the case when "the process of strong earthquake preparation" exists actually. Indeed, if an actual process of earthquake preparation is under way, we can reveal its new features and as the amount of data grows and research progresses the prognosis will become more and more exact. In the terms of the model described above, possibilities of improvement of prognosis are limited from the very beginning owing to random character of earthquake occurrence.

[19] Note that at present the earthquake prediction takes place in statistical sense but it is borne in mind that the reliability of prediction can be improved very significantly with progress in seismology. The model described above suggests that these hopes may by unjustified and an alternative situation without an essential progress can take place.

Modeling the Mechanism of Power Distribution of a Number of Deposits With Different Amount of Mineral Resources

[20] Let us consider the formation of empirical distribution of a number of deposits from the amount of reserves. High concentration of reserves in a few largest deposits is known to be typical of ore and hydrocarbon deposits. Large hydrocarbon deposits [*Burshtein,* 2006; *Kontorovich et al.,* 1985; *Rodkin,* 2006; *Turcotte,* 1997; and others] are known and the large ore deposits are expected [*Burshtein,* 2006; *Laverov and Rundqvist,* 2004, 2006; *Turcotte,* 1997; and others] to obey the power law. That means that the number of deposits N with the stocks amount of no less than V obeys the relation (8) similar to earthquake number distribution in relation to energy or seismic moment:

$$N(V) \cong KV^{-\beta} \ , \tag{8}$$

where K is a coefficient; β is a power exponent of the distribution, in the most cases the meaning of β is close to one. For ore deposits, relation (8) is fulfilled for large and super large deposits only [*Turcotte,* 1997]. As an example of such approximation in Figure 3 the distribution of reserves of 8 lead-zinc deposits largest in the world is given using [*Laverov*

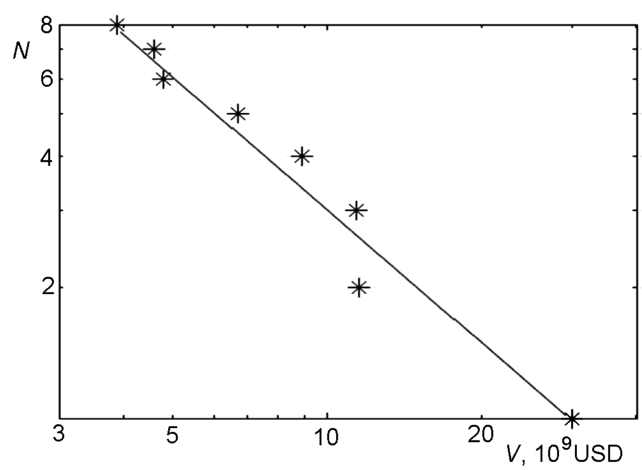

Figure 3. Distribution of total reserves of 8 largest world Pb-Zn deposits (the reserves V values are given in billion dollars). It can be seen that empirical data are well-described by power law with power exponent β close to 1, data from [*Laverov and Rundqvist,* 2006].

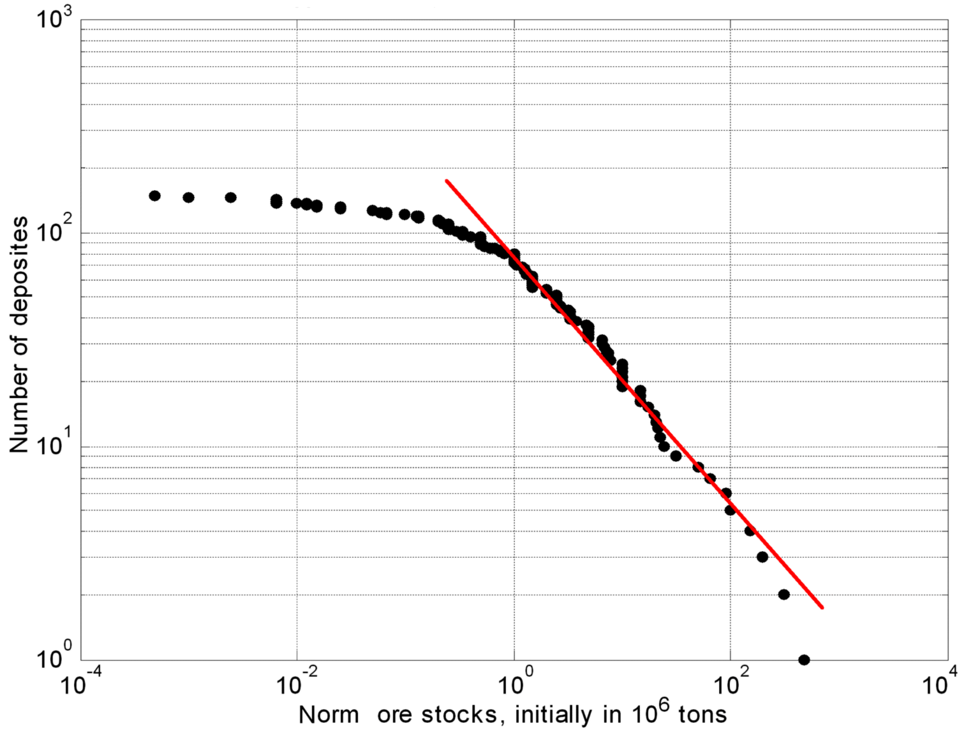

Figure 4. Empirical distribution of normalized values of reserves in carbonatite from data [*Frolov et al.,* 2005]. The major part of distribution (for large deposits) may be described by power law distribution (red line).

and Rundqvist, 2006] data. Taking into account the problem of estimating of the total amount of reserves in case of multi-component deposits the reserve amounts are given in value terms (billion dollars).

[21] In Figure 4, data on ore reserves in carbonatite rocks are given using data [*Frolov et al.,* 2005]. Data on reserves (million tons) for different kinds of mineral raw material are used, these values are standardized by median for the given type of reserves, and data for different ores are combined further to obtain a totality of normalized reserve amounts. The obtained total distribution in the area of large (normalized) values of mineral resources obeys the power distribution law.

[22] For hydrocarbon deposits the relationship (8) is fulfilled reasonably well for mean size deposits besides the large size deposits. Therefore the power distribution law is often applied in practice to estimate the number of undiscovered deposits of a given size in the region under study [*Burshtein,* 2006; *Kontorovich et al.,* 1985].

[23] Let us consider the possibility of modeling the process of origin of power-law‘ relationship (8) for hydrocarbon deposits. Definite data testify for the geological "youth" of the most oil and gas deposits [*Muslimov,* 2005, 2006; and others]. Moreover the effect of recent replacement of hydrocarbon reserves was revealed, and available data suggest a specific law of replacing of hydrocarbon reserves, i.e., the replacement rate is found to be approximately proportional to hydrocarbon total reserves in the deposit [*Rodkin,* 2002a, 2006]. It can be seen easily that such a regime of reserve replacement is in agreement with an avalanche-like process model (1). Such an agreement testifies for the usefulness of model (1) in description of processes of formation and replacement of oil and gas deposits.

[24] In other cases and specifically for presumably long processes of formation of ore deposits the model of deposit formation in accordance with an avalanche-like scheme appears questionable. Actually, the process of ore deposit formation is treated generally as a long-term process and not avalanche-like. Note however, that this concept is not generally accepted; in paper [*Romaniuk and Tkachev,* 2007] geological short-term simultaneous processes of large endogenous ore deposits are supported. From this it follows that different models of ore deposit formation appear to be possible.

[25] A likely variant of accommodating of a positive feedback (1) scheme for the case of ore deposits formation may be related to the model of deposit formation by transmagmatic fluid flows [*Zotov,* 1989; and others]. In this case the appearance of positive feedback can be associated with the fact that the current volume of ore deposit is determined by the volume of discharged transmagmatic fluid flow but the same flow (because of high heat content) traveling through the solidified magma sustains the channel suitable for transport of new portions of deep fluid and ore components contained in it. Thus a positive feedback can occur.

[26] However, keeping in mind a vagueness in deposit formation regime, let us consider possible alternatives to avalanche-like model of formation of power-law distribution (1). Differences in the duration of process of deposit for-

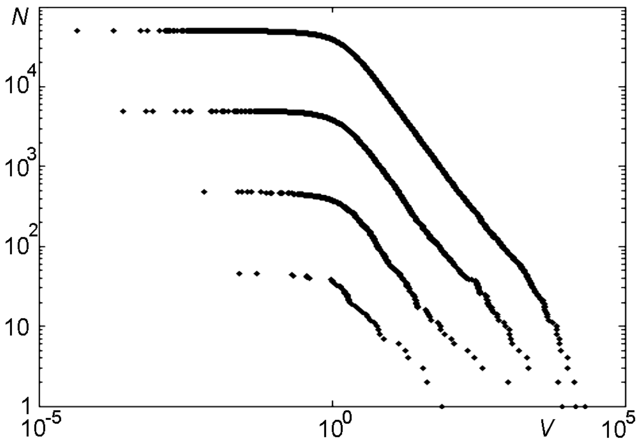

Figure 5. Model distributions of the deposit amount against the amount of reserves for K number of episodes of deposit formation $K = 100$, 1000, 10,000 and 1,000,000. Initial reserves equal to zero, probability of deposit formation process continuation at each step is $p = 0.5$, power parameter value $\beta = 1$. Axis x is amount of reserves V, axis y is the number of deposits N. It can be seen that the number of deposits with smaller amount of reserves is less than their number expected from a power law distribution.

mation may be suggested as an alternative. Indeed, indications to relatively longer duration of process of formation of large and super large earthquakes were found [*Laverov and Rundqvist,* 2004]. However with such a mechanism to accommodate power distribution, the typical duration of formation of smaller, medium and large deposits should differ very strongly. With comparable intensity of deposit formation rates, differences in the duration of forming of smaller deposits and super large deposits would be 10^3–10^4 as much, and to form largest deposits the whole time of existence of the Earth would not suffice. Empirical data in favor of so great differences in the formation time of deposits of different stock values are missing.

[27] The only characteristic that in the context of the used simple model may lead to the required power distribution of a number of deposits in relation to stock values is a power distribution of rates of resource accumulation values. Such distribution seems possible if the formation of endogenous deposits is associated with fluid-magmatic flows. Intensities of these flows are determined by features of corresponding magmatic diapirs and intrusions as well as permeability of faults associated with magma and deep fluid discharge. The structure of tectonic dislocations and diapirs is suggested to be self-similar and hierarchic. Thus, it seems reasonable to suggest that fluid flows corresponding to such structures are self-similar and hierarchic also.

[28] Let us consider a very simple stochastic model of such a process. Suppose that the process of reserve accumulation in an ore deposit might continue with probability of p or cease with probability of $(1 - p)$ at any step. The average rate of resource accumulation for each deposit is suggested to correspond to the value of random Y_i being member of a power-law distribution with typical parameter value $\beta = 1$. We preset the growth of reserves of i-deposit X_i at each subsequent step as the product of mean intensity value of endogenous flow Y_i fixed for each deposit and the random value r, for example evenly distributed in interval [0, 1]. It is reasonable to take the initial amount of reserves in deposit X_0 equal to zero. Assuming the probability of continuing of process of reserve replacement is $p = 0.5$, we obtain at each step

$$\begin{aligned} &\text{If } r_1 > 0.5, \text{ then} \\ &X_i = X_i + r_2 \times Y_i \; . \end{aligned} \quad (9)$$

If it appears that $r_1 < 0.5$, then the formation of this deposit stops and value X_{i-1} as it has been formed in the preceding time moment is taken as the amount of reserves of the given deposit.

[29] In the used simple model, the power law distribution of the mean rates of filling of the given deposit simulates the well-known hierarchic character of the geological discontinuances, for example hierarchic character of fault system being channel for the deposit feeding deep fluid flows. And the random character of a filling process expressed by the product of mean rate and random value r simulates the stochastic character of operating of fluid-magmatic system feeding the deposit. Both these conditions are reasonable and necessary. Therefore model (9) seems to be extremely simple, realistic and structurally stable (from the viewpoint of [*Arnold,* 1998] that models are called stable if variation of characteristics involved changes the details of their behavior without a change in principal features of their behavior). Taking into an account such stability of the model and the reasonable character of assumptions underlying it, one can hope that the major features of behavior of the model are adequate to the major regularities in process of deposits formation.

[30] In Figure 5, curves are given of the distribution of obtained model values of reserves X for different number K of episodes of formation of deposits: $K = 100$, 1000, 10,000, 100,000. Since the probability of continuing of process of deposit formation at each step is $p = 0.5$, actually the process starts in a half of the cases only. It follows that the number of deposits having formed will be close to 50, 500, 5000 and 50,000 respectively. It can be seen that the obtained distributions are similar, and power law character of distribution (with exponent β close to 1) takes place for deposits of larger volume of reserves only. For deposits with smaller reserves volumes the curve deviates from the power-law distribution curve towards considerably less amount of deposits. It should be emphasized that this feature of distribution is a basic feature of the model and it cannot be removed by an insignificant change in parameters of the model.

[31] Similar result of a relatively smaller number of relatively smaller events follows also from the Gnedenko–Pikands –Balkem-de Haan limit theorem of the theory of probability (it is described in more details in [*Embrechts et al.,* 1997; *Pisarenko and Rodkin,* 2007]). Simplifying strict formulations, this theorem states that the distribution of maximum values of excess function of any empirical distribution corresponds to the generalized Pareto distribution (GPD) corre-

sponding to the power law distribution in the area of maximum values and deviating from it in the area of moderate values. The general pattern of GPD distribution is similar to model curves presented in Figure 4, thus the general Gnedenko–Pikands–Balkem-de Haan limit theorem of the theory of probability also indicates a relatively smaller number of deposits with smaller stocks values than it should follow from the clear power-law distribution.

[32] The described above model feature – relatively smaller number of deposits with relatively smaller amount of reserves than it may be expected from the power law distribution – is a feature typical of empirical data [*Burshtein,* 2006; *Kontorovich et al.,* 1985; *Laverov and Rundqvist,* 2004]. This feature is suggested ordinary to arise from the fact that it is more difficult to reveal deposits of smaller amount of resources. The assumption of lower probability of revealing of smaller deposits is feasible but the model used above suggests also another explanation of this phenomenon. The actual number of smaller deposits may be smaller than it should be assumed from the pure distribution power law.

[33] To check this assumption we use data sampling from [*Burshtein,* 2006] on 18 oil basins of North America with a large number of oil deposits. In this paper the actual numbers of deposits with reserve amount of more than and less than 5 million tons and the expected numbers of deposits obtained by the extrapolation of the power law distribution are given. Besides the data on total reserves, age of sedimentary basins and the size of the basins (the areas of sedimentary basins and sedimentary rocks volumes) are presented.

[34] It is reasonable to assume that the relation of actual numbers of oil deposits to the suggested numbers of deposits (theoretically expected number) will increase as the studies in the basin progress and this regularity will be manifested for both larger and smaller deposits (and even somewhat better for smaller deposits owing to a large number of such deposits and statistically more sufficient data).

[35] Having no direct data on the extent of previous studies, we use indirect assessments. Suppose that the extent of previous studies is better in basins with larger number of revealed deposits per area of the basin (N/S) and in basins with larger value of revealed reserves (W). Let us characterize the extent of coverage by value WN/S. Let us compare the amount of revealed deposits of volume greater than 5 million tons (points) and of volume smaller than 5 million tons (circles) to the expected number of such deposits from the model of pure power-like distribution (Figure 6). It can be seen that the share of revealed large deposits shows a tendency of increase with WN/S increasing. However for deposits of smaller volumes such regularity is not noted. The lack of it is corroborated with formal calculations of regression parameters and results obtained with the use of other ways of estimating of comparative extent of coverage in different sedimentary basins. Thus the studies of hydrocarbon deposit data of North America corroborate the conclusion inferred from the model that the number of deposits with relatively smaller volumes of reserves is actually relatively smaller as compared to a power law distribution.

[36] Let us consider statistical behavior of deposits at the stage of "ageing" that corresponds to a stage of failure and disintegration of deposit. Let us consider two variants of disintegration of hydrocarbon deposits with time. The rate of disintegration can be determined by the parameters of feeding fault system Y_i (similar to process of deposit formation; in this case faults by which the stocks are carried out from the deposit are considered to be a continuation of faults that feed it). According to the second variant the rate of deposit disintegration is determined by the amount of its current stock values X_i. Suppose the mechanism of useful raw material loss at each stage is proportional to either X_i or Y_i. We take the rate of deposit disintegration 100 times as slow as the rate of its accumulation. This assumption seems likely and it is intended to model the presence of isolating seals in the near-surface sedimentary layers and the decrease in solubility at lower temperatures in near-surface horizons. Modeling typical result for the case of disintegration rate dependence of effective permeability of feeding system Y_i is given in Figure 7a for times exceeding the deposit formation time by factor of 3, 50 and 500. Similar plot for the case when rate of deposit disintegration is assumed to be proportional to current stock values is shown in Figure 7b.

[37] As it can be seen in Figure 7, in the model used the power character of deposits number distribution against the amount of reserves preserves in a process of deposits disintegration. Distribution plot retains its form and the slope of the plot changes insignificantly and irregularly and the number of deposits of different size linearly decreases with time. Thus within the model under discussion, the power law distribution of a number of deposits from their stock values persists at a very long stage of deposit degradation after the process of deposit formation was completed.

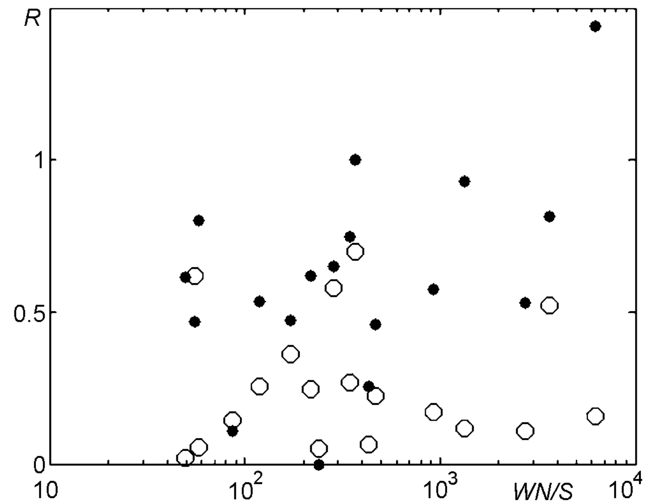

Figure 6. The ratio R of the number of known deposits and their expected number in relation to extent of geological prospecting in the basin estimated by value WN/S for deposits of volume more (points) and less (circles) that 5 billion tons. It can be seen that better quality of prospecting in the basin is not accompanied by the increase of share R of revealed smaller deposits whereas share of large deposits increases, approaching the theoretical number.

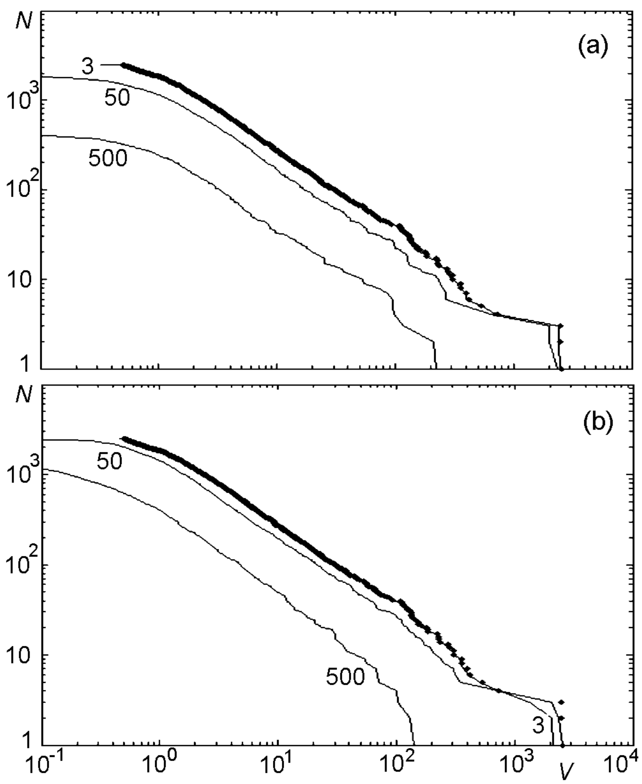

Figure 7. Model of deposit aging, when the rate of deposits disintegration is determined by effective permeability of the corresponding fault system (a) or by current stock values of the deposit (b). Points show initial distribution of reserves before the beginning of deposit disintegration. Lines show model distributions for time intervals exceeding typical time of deposit formation by factor 3, 50 or 5000 (lines are marked with respective figures 3, 50 and 500). In the both variants of the model, the initial power distribution is preserved and the number of deposits of a given value diminishes proportionally to time. Axis x is reserve volume V and axis y is the number of deposits N.

Conclusion

[38] Several simple statistical variants of origin of empirical power distribution typical of earthquake statistics (Gutenberg-Richter law) and of the distribution of a number of mineral deposits in relation to the amount of reserves are examined. To interpret the origin of a power-law distribution in seismology and in the statistics of hydrocarbon deposits the model of generation of such distribution in result of a number of processes of avalanche-like relaxation of meta-stable states is suggested. To simulate processes of ore deposit formation we consider as a basic variant a model based on the assumption of a power-law distribution of intensity of fluid and magmatic flows feeding the deposits.

[39] For the case of examination of earthquakes statistics, it is shown that the effects commonly treated as precursor anomalies may occur in a model with independent events; thus it can occur in the case when precursor effects strictly speaking are impossible. The appearance of such anomalies is caused by correlation of probability of strong event occurrence with change in the regime of a weak seismicity. Treating such features as precursor anomalies is not correct because such anomalies occur not only before strong events but also during some time after them.

[40] The obtained result allows us to propose a variant bringing to agreement the two major alternative approaches to earthquake prognosis problem. According to the first of them [*Geller et al.,* 1997; *Wyss et al.,* 1997; and others], seismicity is believed to be unpredictable in principle. According to the second [*Kosobokov,* 2005; *Shebalin,* 2006; *Sobolev,* 1993; *Sobolev and Ponomarev,* 2003; *Zavialov,* 2006; and others], a definite prognostic behavior in a number of effects that allow to accomplish a statistically non-trivial prognosis of strong earthquakes does take place. The random character of seismic process and even the absence of earthquake preparation process as well as the possibility of obtaining of a non-trivial statistical prognosis of strong earthquakes are retained in the described-above model. Note that this approach is a further development of concepts of limited possibilities of the theory and practice of earthquake prediction. [*Dubois and Gvishiani,* 1998; *Gvishiani and Dubois,* 2002; *Gvishiani and Gurvich,* 1992; and others].

[41] As for the problem of explaining of empirically observed distribution of deposits number in relation to their stock values the model is proposed that explains the fulfillment of the power law distribution of a number of large deposits both at the stage of formation and at the stage of degradation of deposits. Besides, this model leads to a conclusion that the number of deposits with small volume of reserves is considerably less than it follows from an extrapolation of the power law distribution. This conclusion is in a good agreement with empirical results. This deviation is commonly treated as a consequence of relatively smaller probability of revealing of smaller deposits. From the used model it may be assumed that the actual number of medium-size and small earthquakes is relatively smaller as well. Note that this conclusion appears to be significant for the strategy of geological prospecting in old and new mining areas.

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